Study Guide for the Advanced Placement Calculus AB Examination

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Introduction

Advanced Placement[1] is a program of college-level courses and examinations that gives high school students the opportunity to receive advanced placement and/or credit in college. The Advanced Placement Calculus AB Exam tests students on introductory differential and integral calculus, covering a full-year college mathematics course.

There are three sections on the AP Calculus AB Examination:
1. Multiple Choice: Part A (25 questions in 45 minutes) - calculators are not allowed
2. Multiple Choice: Part B (15 questions in 45 minutes) - graphing calculators are required for some questions
3. Free response (6 questions in 45 minutes) - graphing calculators are required for some questions

Scoring

Both sections (multiple choice and free response) are given equal weight.
Grades are reported on a 1 to 5 scale:

	Examination Grade
Extremely well qualified	5
Well qualified	4
Qualified	3
Possibly qualified	2
No recommendation	1

To obtain a grade of 3 or higher, you need to answer about 50 percent of the multiple-choice questions correctly and do acceptable work on the free-response section. In both Parts A and B of the multiple choice section, 1/4 of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly.

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Topics to Study

Elementary Functions

Properties of Functions

A <u>function</u> f is defined as a set of all ordered pairs (x, y), such that for each element x, there corresponds exactly one element y.

The <u>domain</u> of f is the set x.

The <u>range</u> of f is the set y.

Combinations of Functions

If $f(x) = 3x + 1$ and $g(x) = x^2 - 1$

a) the sum $f(x) + g(x) = (3x + 1) + (x^2 - 1) = x^2 + 3x$

b) the difference $f(x) - g(x) = (3x + 1) - (x^2 - 1) = -x^2 + 3x + 2$

c) the product $f(x)g(x) = (3x + 1)(x^2 - 1) = 3x^3 + x^2 - 3x - 1$

d) the quotient $f(x)/g(x) = (3x + 1)/(x^2 - 1)$

e) the composite $(f \circ g)(x) = f(g(x)) = 3(x^2 - 1) + 1 = 3x^2 - 2$

Inverse Functions

Functions f and g are inverses of each other if

$f(g(x)) = x$ for each x in the domain of g

$g(f(x)) = x$ for each x in the domain of f

The inverse of the function f is denoted f^{-1}.

To find f^{-1}, switch x and y in the original equation and solve the equation for y in terms of x.

<u>Exercise</u>: If $f(x) = 3x + 2$, then $f^{-1}(x) =$

(A) $\frac{1}{3x+2}$

(B) $\frac{x}{3} - 2$

(C) $3x - 2$

(D) $\frac{1}{2}x + 3$

(E) $\frac{x-2}{3}$

The answer is E. $x = 3y + 2$

$3y = x - 2$

$y = \frac{x-2}{3}$

Even and Odd Functions

The function $y = f(x)$ is <u>even</u> if $f(-x) = f(x)$.

Even functions are symmetric about the y-axis (e.g. $y = x^2$)

The function $y = f(x)$ is <u>odd</u> if $f(-x) = -f(x)$.

Odd functions are symmetric about the origin (e.g. $y = x^3$)

Exercise: If the graph of $y = 3^x + 1$ is reflected about the y-axis, then an equation of the reflection is $y =$

(A) $3^x - 1$
(B) $\log_3 (x - 1)$
(C) $\log_3 (x + 1)$
(D) $3^{-x} + 1$
(E) $1 - 3^x$

The answer is D. The reflection of $y = f(x)$ in the y-axis is $y = f(-x)$

Periodic Functions

You should be familiar with the definitions and graphs of these trigonometric functions: sine, cosine, tangent, cotangent, secant, and cosecant

Exercise: If $f(x) = \sin(\tan^{-1} x)$, what is the range of f?

(A) $(-p/2, p/2)$
(B) $[-p/2, p/2]$
(C) $(0, 1]$
(D) $(-1, 1)$
(E) $[-1, 1]$

The answer is D. The range of $\sin x$ is (E), but the points at which $\sin x = \pm 1$ ($p/2 + \mathrm{k}p$), $\tan^{-1} x$ is undefined. Therefore, the endpoints are not included.

Note: The range is expressed using interval notation:

$$(a,b) \Leftrightarrow a < x < b$$
$$[a,b] \Leftrightarrow a \le x \le b$$

Zeros of a Function

These occur where the function $f(x)$ crosses the x-axis. These points are also called the roots of a function.

Exercise: The zeros of $f(x) = x^3 - 2x^2 + x$ is

(A) 0, -1
(B) 0, 1
(C) -1
(D) 1
(E) -1, 1

The answer is B. $f(x) = x(x^2 - 2x + 1) = x(x - 1)^2$

Properties of Graphs
You should review the following topics:
a) Intercepts
b) Symmetry
c) Asymptotes
d) Relationships between the graph of
$y = f(x)$ and $y = kf(x)$
$y = f(kx)$
$y - k = f(x - h)$
$y = |f(x)|$
$y = f(|x|)$

Limits

Properties of Limits
If b and c are real numbers, n is a positive integer, and the functions f and g have limits as $x \to c$, then the following properties are true.

1. Scalar multiple: $\lim_{x\to c}[b(f(x))] = b[\lim_{x\to c} f(x)]$
2. Sum or difference: $\lim_{x\to c}[f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$
3. Product: $\lim_{x\to c}[f(x)g(x)] = [\lim_{x\to c} f(x)][\lim_{x\to c} g(x)]$
4. Quotient: $\lim_{x\to c}[f(x)/g(x)] = [\lim_{x\to c} f(x)]/[\lim_{x\to c} g(x)]$, if $\lim_{x\to c} g(x) \neq 0$

One-Sided Limits
$\lim_{x\to a+} f(x)$ x approaches c from the right
$\lim_{x\to a-} f(x)$ x approaches c from the left

Limits at Infinity
$\lim_{x\to +\infty} f(x) = L$ or $\lim_{x\to -\infty} f(x) = L$
The value of $f(x)$ approaches L as x increases/decreases without bound.
$y = L$ is the <u>horizontal asymptote</u> of the graph of f.

Some Nonexistent Limits
$\lim_{x\to 0} \frac{1}{x^2}$ $\lim_{x\to 0} \frac{|x|}{x}$ $\lim_{x\to 0} \sin\frac{1}{x}$

Some Infinite Limits
$\lim_{x\to 0} \frac{1}{x^2} = \infty$ $\lim_{x\to 0+} \ln x = -\infty$

Exercise: What is $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$?

(A) 1
(B) 0
(C) ∞
(D) $\frac{p}{2}$
(E) The limit does not exist.

The answer is A. You should memorize this limit.

Continuity

Definition

A function f is continuous at c if:

1. $f(c)$ is defined
2. $\lim\limits_{x \to c} f(x)$ exists
3. $\lim\limits_{x \to c} f(x) = f(c)$

Graphically, the function is continuous at c if a pencil can be moved along the graph of $f(x)$ through $(c, f(c))$ without lifting it off the graph.

Exercise:

$$\text{If} \quad \begin{cases} f(x) = \dfrac{3x^2 + x}{2x} & \text{for } x \neq 0 \\ f(0) = k, & \end{cases}$$

and if f is continuous at $x = 0$, then $k =$

(A) -3/2
(B) -1
(C) 0
(D) 1
(E) 3/2

The answer is E. $\lim\limits_{x \to 0} f(x) = 3/2$

Intermediate Value Theorem

If f is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c between a and b such that $f(c) = k$.

Differential Calculus

Definition

$f'(x) = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

and if this limit exists

$f'(c) = \lim_{x \to c} \dfrac{f(x) - f(c)}{x - c}$

If f is differentiable at $x = c$, then f is continuous at $x = c$.

Differentiation Rules

General and Logarithmic Differentiation Rules

1. $\dfrac{d}{dx}[cu] = cu'$
2. $\dfrac{d}{dx}[u \pm v] = u' \pm v'$ sum rule
3. $\dfrac{d}{dx}[uv] = uv' + vu'$ product rule
4. $\dfrac{d}{dx}[\dfrac{u}{v}] = \dfrac{vu' - uv'}{v^2}$ quotient rule
5. $\dfrac{d}{dx}[c] = 0$
6. $\dfrac{d}{dx}[u^n] = nu^{n-1}u'$ power rule
7. $\dfrac{d}{dx}[x] = 1$
8. $\dfrac{d}{dx}[\ln u] = \dfrac{u'}{u}$
9. $\dfrac{d}{dx}[e^u] = e^u u'$
10. $\dfrac{d}{dx}[f(g(x))] = f'(g(x))\, g'(x)$ chain rule

Derivatives of the Trigonometric Functions

1. $\dfrac{d}{dx}[\sin u] = (\cos u)u'$
2. $\dfrac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
3. $\dfrac{d}{dx}[\cos u] = -(\sin u)u'$
4. $\dfrac{d}{dx}[\sec u] = (\sec u \tan u)u'$
5. $\dfrac{d}{dx}[\tan u] = (\sec^2 u)u'$
6. $\dfrac{d}{dx}[\cot u] = -(\csc^2 u)u'$

Derivatives of the Inverse Trigonometric Functions

1. $\dfrac{d}{dx}[\arcsin u] = \dfrac{u'}{\sqrt{1-u^2}}$
2. $\dfrac{d}{dx}[\text{arccsc } u] = \dfrac{-u'}{|u|\sqrt{u^2-1}}$
3. $\dfrac{d}{dx}[\arccos u] = \dfrac{-u'}{\sqrt{1-u^2}}$
4. $\dfrac{d}{dx}[\text{arcsec } u] = \dfrac{u'}{|u|\sqrt{u^2-1}}$
5. $\dfrac{d}{dx}[\arctan u] = \dfrac{u'}{1+u^2}$
6. $\dfrac{d}{dx}[\text{arccot } u] = \dfrac{-u'}{1+u^2}$

Implicit Differentiation

Implicit differentiation is useful in cases in which you cannot easily solve for y as a function of x.

Exercise: Find $\frac{dy}{dx}$ for $y^3 + xy - 2y - x^2 = -2$

$$\frac{dy}{dx}\,[y^3 + xy - 2y - x^2] = \frac{dy}{dx}\,[-2]$$
$$3y^2\frac{dy}{dx} + (x\frac{dy}{dx} + y) - 2\frac{dy}{dx} - 2x = 0$$
$$\frac{dy}{dx}(3y^2 + x - 2) = 2x - y$$
$$\frac{dy}{dx} = \frac{2x - y}{3y^2 + x - 2}$$

Higher Order Derivatives

These are successive derivatives of $f(x)$. Using prime notation, the second derivative of $f(x)$, $f''(x)$, is the derivative of $f'(x)$. The numerical notation for higher order derivatives is represented by:

$$f^{(n)}(x) = y^{(n)}$$

The second derivative is also indicated by $\frac{d^2y}{dx^2}$.

Exercise: Find the third derivative of $y = x^5$.

$$y' = 5x^4$$
$$y'' = 20x^3$$
$$y''' = 60x^2$$

Derivatives of Inverse Functions

If $y = f(x)$ and $x = f^{-1}(y)$ are differentiable inverse functions, then their derivatives are reciprocals:

$$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

Logarithmic Differentiation

It is often advantageous to use logarithms to differentiate certain functions.

1. Take ln of both sides
2. Differentiate
3. Solve for y'
4. Substitute for y
5. Simplify

Exercise: Find $\frac{dy}{dx}$ for $y = \left(\frac{x^2 + 1}{x^2 - 1}\right)^{1/3}$

$$\ln y = \frac{1}{3}[\ln(x^2 + 1) - \ln(x^2 - 1)]$$
$$\frac{y'}{y} = \frac{1}{3}\left[\frac{1}{x^2 + 1} - \frac{1}{x^2 - 1}\right]$$

$$y' = \frac{-2}{(x^2+1)(x^2-1)}\left(\frac{x^2-1}{x^2+1}\right)^{1/3}$$

$$y' = \frac{-2}{(x^2+1)^{4/3}(x^2-1)^{2/3}}$$

Mean Value Theorem

If f is continuous on [a, b] and differentiable on (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

L'Hôpital's Rule

If lim $f(x)/g(x)$ is an indeterminate of the form 0/0 or ∞/∞, and if lim $f'(x)/g'(x)$ exists, then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$$

The indeterminate form $0 \cdot \infty$ can be reduced to 0/0 or ∞/∞ so that L'Hôpital's Rule can be applied.

<u>Note</u>: L'Hôpital's Rule can be applied to the four different indeterminate forms of ∞/∞: ∞/∞, $(-\infty)/\infty$, $\infty/(-\infty)$, and $(-\infty)/(-\infty)$

<u>Exercise</u>: What is $\lim_{x\to 0} \frac{\sin x + 1}{x}$?

(A) 2
(B) 1
(C) 0
(D) ∞
(E) The limit does not exist.

The answer is B. $\lim_{x\to 0} \frac{\cos x}{1} = 1$

Tangent and Normal Lines

The derivative of a function at a point is the slope of the <u>tangent line</u>. The <u>normal line</u> is the line that is perpendicular to the tangent line at the point of tangency.

<u>Exercise</u>: The slope of the normal line to the curve $y = 2x^2 + 1$ at (1, 3) is

(A) -1/12
(B) -1/4
(C) 1/12
(D) 1/4
(E) 4

The answer is B. $y' = 4x$
$y = 4(1) = 4$
slope of normal = -1/4

Extreme Value Theorem
If a function $f(x)$ is continuous on a closed interval, then $f(x)$ has both a maximum and minimum value in the interval.

Curve Sketching

Situation	Indicates
$f'(c) > 0$	f increasing at c
$f'(c) < 0$	f decreasing at c
$f'(c) = 0$	horizontal tangent at c
$f'(c) = 0,\ f'(c^-) < 0,\ f'(c^+) > 0$	relative minimum at c
$f'(c) = 0,\ f'(c^-) > 0,\ f'(c^+) < 0$	relative maximum at c
$f'(c) = 0,\ f''(c) > 0$	relative minimum at c
$f'(c) = 0,\ f''(c) < 0$	relative maximum at c
$f'(c) = 0,\ f''(c) = 0$	*further investigation required*
$f''(c) > 0$	concave upward
$f''(c) < 0$	concave downward
$f''(c) = 0$	*further investigation required*
$f''(c) = 0,\ f''(c^-) < 0,\ f''(c^+) > 0$	point of inflection
$f''(c) = 0,\ f''(c^-) > 0,\ f''(c^+) < 0$	point of inflection
$f(c)$ exists, $f'(c)$ does not exist	possibly a vertical tangent; possibly an absolute max. or min.

Newton's Method for Approximating Zeros of a Function

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

To use Newton's Method, let x_1 be a guess for one of the roots. Reiterate the function with the result until the required accuracy is obtained.

Optimization Problems
Calculus can be used to solve practical problems requiring maximum or minimum values.

Exercise: A rectangular box with a square base and no top has a volume of 500 cubic inches. Find the dimensions for the box that require the least amount of material.

Let V = volume, S = surface area, x = length of base, and h = height of box

$V = x^2h = 500$
$S = x^2 + 4xh = x^2 + 4x(500/x^2) = x^2 + (2000/x)$
$S' = 2x - (2000/x^2) = 0$
$2x^3 = 2000$

$x = 10$, $h = 5$
Dimensions: 10 x 10 x 5 inches

Rates-of-Change Problems

Distance, Velocity, and Acceleration

$y = s(t)$ — position of a particle along a line at time t

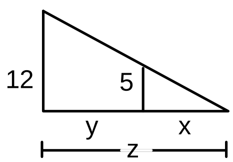
$v = s'(t)$ — instantaneous velocity (rate of change) at time t

$a = v'(t) = s''(t)$ — instantaneous acceleration at time t

Related Rates of Change

Calculus can be used to find the rate of change of two or more variable that are functions of time t by differentiating with respect to t.

Exercise: A boy 5 feet tall walks at a rate of 3 feet/sec toward a streetlamp that is 12 feet above the ground.
a) What is the rate of change of the tip of his shadow?
b) What is the rate of change of the length of his shadow?



$\frac{5}{x} = \frac{12}{z}$ $\qquad$ $\frac{x+y}{x} = \frac{12}{5}$

$z = \frac{12}{5}x$ $\qquad$ $x = \frac{5}{7}y$

$\frac{dx}{dt} = \frac{5}{7}\left(\frac{dy}{dt}\right)$ $\qquad$ $\frac{dz}{dt} = \frac{12}{5}\left(\frac{dx}{dt}\right)$

$\frac{dx}{dt} = \frac{5}{7}(3)$ $\qquad$ $\frac{dz}{dt} = \frac{12}{5}\left(\frac{15}{7}\right)$

b) $= \frac{15}{7}$ ft/sec $\qquad$ a) $= \frac{36}{7}$ ft/sec

Note: the answers are independent of the distance from the light.

Exercise: A conical tank 20 feet in diameter and 30 feet tall (with vertex down) leaks water at a rate of 5 cubic feet per hour. At what rate is the water level dropping when the water is 15 feet deep?

$V = \frac{1}{3} p\, r^2 h$ $\qquad$ $\frac{dv}{dt} = \frac{1}{9} p\, h^2 \frac{dh}{dt}$

$\frac{r}{h} = \frac{10}{30}$ $\qquad$ $5 = \frac{1}{9} p\, h^2 \frac{dh}{dt}$

$r = \frac{1}{3}h$ $\qquad$ $\frac{dh}{dt} = \frac{45}{\pi h^2}$

$V = \frac{1}{27} p\, h^3$ $\qquad$ $\frac{dh}{dt} = \frac{1}{5\pi}$ ft/hr

Integral Calculus

Indefinite Integrals

Definition: A function $F(x)$ is the antiderivative of a function $f(x)$ if for all x in the domain of f,

$$F'(x) = f(x)$$

$$\int f(x)\,dx = F(x) + C, \text{ where } C \text{ is a constant.}$$

Basic Integration Formulas

General and Logarithmic Integrals

1. $kf(x)\,dx = k\,f(x)\,dx$
2. $\int [f(x) \pm g(x)]\,dx = f(x)\,dx \pm g(x)\,dx$
3. $\int k\,dx = kx + C$
4. $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C,\ n \neq -1$
5. $\int e^x\,dx = e^x + C$
6. $\int a^x\,dx = \dfrac{a^x}{\ln a} + C,\ a > 0,\ a \neq 1$
7. $\int \dfrac{dx}{x} = \ln |x| + C$

Trigonometric Integrals

1. $\int \sin x\,dx = -\cos x + C$
2. $\int \cos x\,dx = \sin x + C$
3. $\int \sec^2 x\,dx = \tan x + C$
4. $\int \csc^2 x\,dx = -\cot x + C$
5. $\int \sec x \tan x\,dx = \sec x + C$
6. $\int \csc x \cot x\,dx = -\csc x + C$
7. $\int \tan x\,dx = -\ln |\cos x| + C$
8. $\int \cot x\,dx = \ln |\sin x| + C$
9. $\int \sec x\,dx = \ln |\sec x + \tan x| + C$
10. $\int \csc x\,dx = -\ln |\csc x + \cot x| + C$
11. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \arcsin \dfrac{x}{a} + C$
12. $\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a} \arctan \dfrac{x}{a} + C$
13. $\int \dfrac{dx}{x\sqrt{x^2 - a^2}} = \dfrac{1}{a} \operatorname{arcsec} \dfrac{x}{a} + C$

Integration by Substitution

$$\int f(g(x))g'(x)\,dx = F(g(x)) + C$$

$$\text{If } u = g(x), \text{ then } du = g'(x)\,dx \text{ and } \int f(u)\,du = F(u) + C$$

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Distance, Velocity, and Acceleration (*on Earth*)

$a(t) = s''(t) = -32 \text{ ft/sec}^2$

$v(t) = s'(t) = \int s''(t)\,dt = \int -32\,dt = -32t + C_1$

at $t = 0$, $v_0 = v(0) = (-32)(0) + C_1 = C_1$

$s(t) = \int v(t)\,dt = \int (-32t + v_0)\,dt = -16t^2 + v_0 t + C_2$

Separable Differential Equations
It is sometimes possible to separate variables and write a differential equation in the form $f(y)\,dy + g(x)\,dx = 0$ by integrating:

$$\int f(y)\,dy + \int g(x)\,dx = C$$

Exercise: Solve for $\frac{dy}{dx} = \frac{-2x}{y}$

$2x\,dx + y\,dy = 0$
$x^2 + \frac{y^2}{2} = C$

Applications to Growth and Decay
Often, the rate of change or a variable y is proportional to the variable itself.

$\frac{dy}{dt} = ky$	separate the variables
$\frac{dy}{y} = k\,dt$	integrate both sides
$\ln\|y\| = kt + C_1$	
$y = Ce^{kt}$	Law of Exponential Growth and Decay Exponential growth when $k > 0$ Exponential decay when $k < 0$

Definition of the Definite Integral
The definite integral is the limit of the Riemann sum of f on the interval $[a, b]$

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(x_i)\Delta x = \int_a^b f(x)\,dx$$

Properties of Definite Integrals
1. $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
2. $\int_a^b kf(x)\,dx + k\int_a^b f(x)\,dx$
3. $\int_a^a f(x)\,dx = 0$
4. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
5. $\int_b^a f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$
6. If $f(x) \le g(x)$ on $[a, b]$, then $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$

Approximations to the Definite Integral

Riemann Sums

$\int_a^b f(x)dx = S_n = \sum_{i=1}^{n} f(x_i)\Delta x$

Trapezoidal Rule

$\int_a^b f(x)dx \approx [\frac{1}{2} f(x_0) + f(x_1) + f(x_2) + ... + f(x_{n-1}) + \frac{1}{2} f(x_n)] \frac{b-a}{n}$

The Fundamental Theorem of Calculus

If f is continuous on $[a, b]$ and if $F' = f$, then

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

The Second Fundamental Theorem of Calculus

If f is continuous on an open interval I containing a, then for every x in the interval,

$$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$$

Area Under a Curve

If	$f(x) \geq 0$ on $[a, b]$	$A = \int_a^b f(x)\, dx$
If	$f(x) \leq 0$ on $[a, b]$	$A = -\int_a^b f(x)\, dx$
If	$f(x) \geq 0$ on $[a, c]$ and $f(x) \leq 0$ on $[c, b]$	$A = \int_a^c f(x)\, dx - \int_c^b f(x)\, dx$

Exercise

The area enclosed by the graphs of $y = 2x^2$ and $y = 4x + 6$ is:

(A) 76/3

(B) 32/3

(C) 80/3

(D) 64/3

(E) 68/3

The answer is D. Intersection of graphs: $2x^2 = 4x + 6$

$2x^2 - 4x + 6 = 0$

$x = -1, 3$

$A = \int_{-1}^{3} 4x + 6 - 2x^2$

$= (2x^2 + 6x - \frac{2x^3}{3})\Big|_{-1}^{3}$

$= 18 + 18 - 18 - (2 - 6 + 2/3)$

$= 64/3$

Average Value of a Function on an Interval

$$\frac{1}{b-a} \int_a^b f(x)\, dx$$

Volumes of Solids with Known Cross Sections

1. For cross sections of area $A(x)$, taken perpendicular to the x-axis:
$V = \int_a^b A(x)\, dx$

2. For cross sections of area $A(y)$, taken perpendicular to the y-axis:
$V = \int_a^b A(y)\, dy$

Volumes of Solids of Revolution: Disk Method

$V = \int_a^b p\, r^2\, dx$

Rotated about the x-axis: $V = \int_a^b p\, [f(x)]^2\, dx$

Rotated about the y-axis: $V = \int_a^b p\, [f(y)]^2\, dy$

Volumes of Solids of Revolution: Washer Method

$V = \int_a^b p\, (r_o^2\, dx - r_i^2\,)\, dx$

Rotated about the x-axis: $V = \int_a^b p\, [(f_1(x))^2 - (f_2(x))^2]\, dx$

Rotated about the y-axis: $V = \int_a^b p\, [(f_1(y))^2 - (f_2(y))^2]\, dy$

<u>Exercise</u>: Find the volume of the region bounded by the y-axis, $y = 4$, and $y = x^2$ if it is rotated about the line $y = 6$.

$$p \int_0^2 [(x^2 - 6)^2 - (4 - 6)^2\,]dx$$
$$= \frac{192p}{5} \text{ cubic units}$$

Volumes of Solids of Revolution: Cylindrical Shell Method

$V = \int_a^b 2p\, rh\, dr$

Rotated about the x-axis: $V = 2p \int_a^b xf(x)\, dx$

Rotated about the y-axis: $V = 2p \int_a^b yf(y)\, dy$

Some Useful Formulas

$\log_a x = \dfrac{\log x}{\log a}$

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$

$\sin 2x = 2 \sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2 x = ½(1- \cos 2q)$
$\cos^2 x = ½(1+ \cos 2q)$

Volume of a right circular cylinder = $p\ r^2 h$
Volume of a cone = $\frac{1}{3}\pi r^2 h$
Volume of a sphere = $\frac{4}{3}\pi r^3$

Calculator Tips and Programs

Your calculator will serve as an extremely useful tool if you take advantage of all of its functions. We will base all of the following tips and programs on the TI-82, which most calculus students use today.

On the AP Calculus Exam, you will need your calculator for Part B of Section I and for Section II. You will need to know how to do the following:

1. simple calculations
2. find the intersection of two graphs
3. graph a function and be able to find properties listed under *Elementary Functions* in the *Topics to Study* section (e.g. domain, range, asymptotes)

Here are some of the functions available that you should know how to use:

In the **CALC** menu:

1. calculate the value of a function at x = c
2. calculate the roots of a function
3. find the minimum of a function
4. find the maximum of a function
5. find the point of intersection of two functions
6. find the slope of the tangent at (x, y)
7. find the area under the curve from a to b

In the **MATH MATH** menu:

6. find the minimum of a function
fMin(*expression***,** *variable***,** *lower***,** *upper***)**
7. find the maximum of a function
fMax(*expression***,** *variable***,** *lower***,** *upper***)**
8. find the numerical derivative at a given value
nDeriv(*expression***,** *variable***,** *value***)**
9. find the numerical integral of an expression
fnInt(*expression***,** *variable***,** *lower***,** *upper***)**
0. calculate the root of an expression
solve(*expression***,** *variable***,** *guess***, {***lower***,** *upper***})**

Calculator Programs

One of the easiest programs to create is one that will solve for f(x). You can also run the program multiple times to find other values for the same function.

```
PROGRAM: SOLVE
: Input X
: 3x^2 + 2 → X          [type your function here and place → X at the end]
: Disp X
```

Here is a program to solve for a quadratic equation:

```
PROGRAM: QUADRAT
:Input "A? ", A
:Input "B? ", B
:Input "C? ", C
:(- B + √ (B²  - 4AC) ) / 2A → D
:(- B - √ (B²  - 4AC) ) / 2A → E
:B²  - 4AC → F
:ClrHome
:Disp "+ EQUALS"
:Disp D
:Disp "- EQUALS"
:Disp E
:Disp "B²  - 4AC EQUALS"
:Disp F
```

To Run: Enter a, b, and c for $ax^2 + bx + c$.
"+ EQUALS" and "- EQUALS" give the roots of the equation

Here is a program that will use the trapezoidal rule to approximate a definite integral:

```
PROGRAM: TRAP
:ClrHome
:Input "F(X) IN QUOTES:", Y0
:Input "START(A):", A
:Input "END(B):", B
:Input "NO. OF DIV. (N):", N
:(B - A) / N → D
:0 → S
:For (X, A, B, D)
:S + Y0 → S
:End
:A → X : Y0 → F
:B → X : Y0 → L
:D * (-F + S - L) → A
:ClrHome
:Disp "EST AREA="
:Disp A
```

Book Review of Available Study Guides

Brook, Donald E., Donna M. Smith, and Tefera Worku. The Best Test Preparation for the Advanced Placement Examination in Calculus AB. Piscataway: Research & Education Association, 1995.

This book contains six full-length AP Exams and a short, comprehensive AP course review. This book is for the student who wishes to practice taking the Calculus AB Exam. The topic review is not very clear, and there are several errors in the questions and keys. However, detailed solutions are presented for all problems.

Hockett, Shirley O. Barron's How to Prepare for the Advanced Placement Examinations, Mathematics. New York: Barron's Educational Series, Inc., 1987.

This book contains a review of calculus topics, practice multiple-choice questions for each unit, and four practice examinations and 3 actual examinations for both the Calculus AB and BC Exams. This book is the most comprehensive study guide that I have found. However, the current edition of this text would be much more useful.

Smith, Sanderson M. and Frank W. Griffin. Advanced Placement Examinations in Mathematics. New York: Simon & Schuster, Inc., 1990.

This book contains a complete review of all exam topics, including multiple-choice and free-response questions with explanations. It also includes two full-length practice tests with explanatory answers and BASIC computer programs to reinforce calculus concepts. This book is also a good way to prepare for the Calculus Exams AB and BC.

Zandy, Bernard V. Cliffs Quick Review Calculus. Lincoln: Cliffs Notes, Inc., 1993.

This book is a compact review of all topics covered in a first-year calculus class. Example problems are given in each unit. This book contains the best topical review that I have found. It was not prepared specifically for the AP Exam, so students will need to review the trapezoidal rule, which was not covered in this book. Students will not need to know the arc length formula for the Calculus AB Exam.

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